SAMPLE PAPER 2008

CBSE CLASS XII BOARD EXAMINATION

Time : 3 hrs | Marks : 100

General Instructions

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no all over choice. However, an internal choice has been produced in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted. You may ask for logarithmic and statistical tables if required.

(vi) If you wish to answer any question already answered for any reason, cancel the previous answer. If not cancelled, the first answer will be considered for checking.

SECTION - A

1. Let $t=\sin\left(\frac{\pi}{3}-\sin^{-1}(-\frac{1}{2})\right)$ then $\tan^{-1}t$ equals?

2. Let $A=\begin{pmatrix} a & \beta \\ y & -\alpha \end{pmatrix}$ such that $A^2 = I_2$ then the value of $2-\alpha^2-\beta y$ equals?

3. $\int 2^x\, 2^{2^x}\, 2^{2^{2^x}}\, dx$ equals?

4. The projection of vector $i + j + k$ on the line whose vector equation is $\vec{r}=(3+t)\hat{i}+(2t-1)\hat{j}+3t\hat{k},(t\in R)$ is m then value of m equals?

5. Is function $y=\frac{1}{(x+6)^3}\ (x\neq -6)$ always increasing?

6. The numbers 1, 2, ..., n are arranged in a random order. What is the probability 1, 2, 3, ... r $(r<80)$ appear as neighbours in that order?

7. If $I=\int_0^1 x(1-x)^{24}dx$ then what will be the value of $\frac{1}{I}$.

8. If α is a non real cube root of –2, then what will be the value of $\begin{vmatrix} 1 & 2\alpha & 1 \\ \alpha^2 & 1 & 3\alpha^2 \\ 2 & 2\alpha & 1 \end{vmatrix}$

9. If the plane has intercept a, b, c and is at a distance m units from the point where the three mutually perpendicular axis meet each other than $a^{-2} + b^{-2} + c^{-2}$ equals?

10. What is the order of the differential equation whose general solution is given by $y = (k_1 + k_2)\cos(x + k_3) + k_4 e^{x + k5}$ where k_1, k_2, k_3, k_4, k_5 are arbitrary constants?

SECTION - B

11. Consider the binary operation '$*$' : $R \times R \rightarrow R$ and '$\circ$' : $R \times R \rightarrow R$ defined by $a*b=|a-b|$ and $a \circ b = a\ \forall\ a,b\in R$ then prove that :

(a) Operation $*$ is commutative but not associative.

(b) Operation $\circ$ is associative but not commutative.

12. Let $x\binom{2}{3}+y\binom{-1}{1}=\binom{10}{5}$ and $A=\begin{pmatrix} 2x & y-2 & \frac{5x}{3} \\ y/4 & 3 & -y \\ x+y & y-x & x-y \end{pmatrix}$ then express A as the sum of symmetric and skew symmetric matrices.

OR

By using the properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^2$$

13. Find the value of

$$\operatorname{cosec}^{-1}\sqrt{5} + \operatorname{cosec}^{-1}\sqrt{65} + \operatorname{cosec}^{-1}\sqrt{325}\ldots.. \text{ to } \infty$$

OR

If $a_1, a_2, a_3, \ldots\ldots a_n$ is an A.P with common difference d then find the value of

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

14. Let $f(x)\begin{cases} \dfrac{1-\cos 4x}{x^2} & \text{if} \quad x<0 \\ a & \text{if} \quad x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if} \quad x>0 \end{cases}$

Determine the value of a, if posssible, so that the function is continuous at $x = 0$.

15. If $\sqrt{\frac{y}{x}}+\sqrt{\frac{x}{y}}=6$ then prove that $\frac{dy}{dx}=\frac{x-17y}{17x-y}$

OR

If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$,

then prove that $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$.

16. Evaluate $\int \frac{\sqrt{\cos\theta}}{\sin\theta}\, d\,\theta$.

17. A ladder 13 m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall at the rate 1.5 m/sec. How fast the angle between the ladder and the ground changes when the foot of the ladder is 12 m away from the wall?

18. Evaluate

$$\lim_{n\to\infty}\frac{2}{n}\left[7n+\frac{18}{n}\frac{n(n-1)}{2}+\frac{8}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

19. Evaluate $\int_0^{3/2} |x\cos\pi x|\,dx$.

20. Show that lines

$\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(-3\hat{i}+\hat{j}+5\hat{k})$ and

$\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(-\hat{i}+2\hat{j}+5\hat{k})$ are coplanar, also write the cartesion form of the lines.

21. Show that points $A(2\hat{i}-\hat{j}+\hat{k}), B(\hat{i}-3\hat{j}-5\hat{k}), C(3i-4\hat{j}-4\hat{k})$ are vertices of a right angle triangle.

22. Six dice are thrown 729 times. How many times do you expect at least three dice to show 5 or 6.

OR

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of missing card to be a heart.

SECTION - C

23. If $A=\begin{pmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{pmatrix}$ are two square matrices, find AB and hence by using AB solve the system of linear equations.

$x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7.$

24. A window of fixed perimeter (including the base of the arc) is in the form of rectangle surmounted by a semi-circle. The semi-circular portion is filled with coloured glass while the rectangular part is filled with clear glass . The clear glass transmits three times as much light per square meter as the coloured glass does. What is the ratio of the sides of the rectangle so that the window transmits the maximum light?

OR

The sum of the suface areas of a sphere and a cube is given , prove that sum of their volumes be least if diameter of sphere is equal to the edge of the cube.

25. The sine and cosine function intersect each other infinitely many times, bounding regions of equal area's. Find the area of one of these region by plotting the graph.

26. Solve the differential equation :

$$\frac{d^2y}{dx^2}=e^x\left(\frac{1}{x}-\frac{2}{x^2}+\frac{2}{x^3}\right)$$

Given that $\frac{dy}{dx}=1$ for $x = 1$ and $y = 1$.

27. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on first class ticket and Rs. 300 on economy ticket. An airline reserves at least 20 seats for first class tickets. However, at least 4 times as many passengers prefer to travel economy class than by the first class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

28. Three coins are tossed simutaneously. Consider the event 'E' three heads or three tails, 'F' at least two heads and 'G' at most two heads of pair (E, F), (E, G), (F, G). Which are independent? Which are dependent?

OR

Let x, denote the number of hours you study during a randomly selected school days. The probability that x can take the values, has the following form, where k is some unknown constant

$$p(X=x)=\begin{cases} 0.1 & \text{if} \quad x=0 \\ kx & \text{if} \quad x=1 \text{ or } 2 \\ k(5-x) & \text{if} \quad x=3 \text{ or } 4 \\ 0, \text{ other wise} \end{cases}$$

(a) Find the value of k?

(b) What is the probability that you study atleast two hours, exactly two hours, at most two hours?

(c) Draw the graph of probability distribution.

29. Find the image of the point 1, 2, 3 in the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$, also obtain the equation of the plane containing the line and the point (1, 2, 3).

ANSWERS

1. $\tan^{-1} t = \frac{\pi}{4}$; **2.** $2 - \alpha^2 - \beta y = 1$

3. $\frac{2^{2^{2x}}}{(\log 2)^3}+k$ **4.** $\frac{6}{\sqrt{14}}$

5. $f(x)$ decreases $\forall\, x \in R-\{6\}$

6. Probability $=\frac{(n-r+1)!}{(n)!}$

7. $\frac{1}{I}=600$ **8.** -13

9. $a^{-2} + b^{-2} + c^{-2} = m^{-2}$

10. $y = A\cos(x + k_3) + Be^x$
Which contains three arbitrary constants. Order of differential is three.

11. (a) '$*$' is not associative.
(b) 'o' is not commutative.
'o' is associative.

12. $$A=\begin{pmatrix} 6 & -\frac{7}{2} & 2 \\ -\frac{7}{2} & 3 & -\frac{3}{2} \\ 2 & -\frac{3}{2} & 7 \end{pmatrix}+\begin{pmatrix} 0 & -\frac{5}{2} & 3 \\ \frac{5}{2} & 0 & \frac{11}{2} \\ -3 & -\frac{11}{2} & 0 \end{pmatrix}$$

OR L.H.S = R.H.S

13. $\frac{\pi}{4}$ OR $\frac{(n-1)d}{1+a_1 a_n}$ **14.** $a = 8$

15. $\frac{dy}{dx}=\frac{17y-x}{y-17x}$ or $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

16. $\tan^{-1}\sqrt{\cos\theta}-\frac{1}{2}\log\frac{1+\sqrt{\cos\theta}}{1-\sqrt{\cos\theta}}+k$

17. $\frac{-3}{10}$ radian/sec. **18.** $\frac{112}{3}$ **19.** $\frac{5}{2\pi}-\frac{1}{\pi^2}$

20. $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$; $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$

21. Triangle is right angle triangle.

22. 233 or $\frac{11}{50}$ **23.** $x = 2$, $y = -1$, $z = 4$

24. $x : y = 6 : \pi + 6$ OR Volume is least when diameter of sphere is equal to edge of cube.

25. $2\sqrt{2}$ square units.

26. $y=\frac{e^x}{x}+x-e$

27. Maximum profit = Rs. 64,000; First class tickets = 40; Economy class tickets = 160.

28. Events E and F are independent and events (E and G), (F and G) are dependent.
OR (a) $k = 0.15$, $P(x = r \le 2)$ 0.55.

29. $18x - 22y + 5z + 11 = 0$.

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